Solutions Midterm MAT 5190 M. Alvo

1. The Jacobian $J = 1$. Hence the joint density of z_1, z_2 for any θ in part b)

$$f(z_1, z_2) = \frac{1}{2\pi} e^{-(x_1^2+y^2)} = \frac{1}{2\pi} e^{-(z_1^2+z_2^2)}$$

Choose $\theta = 45°$ for part a)

2. $X_{(1)} - X_{(n)}$ is ancillary, $\bar{X}$ is sufficient and complete for μ. Hence by Basu's Theorem they are indep.

3. a) Write $f(\underline{x}) = h(\underline{x})\, e^{\sum_1^{k-1} x_i \log(p_i/p_k)}\; e^{n \log p_k}$

Consequently $(X_1, .., X_{k-1})$ is minimal sufficient

b) Set $\eta = \log \frac{p}{1-p}$ $\quad t(x) = x$, $\psi(\eta) = -n \log(1-p)$

$$\therefore E\, t(X) = \frac{\partial \psi}{\partial \eta} = -\frac{n}{1-p}\left(-\frac{\partial p}{\partial \eta}\right) = \frac{n}{1-p} \frac{1}{(\partial \eta / \partial p)}$$

$$= \frac{n}{1-p}\, p(1-p) = np$$

$$\text{Var}\, t(X) = \frac{\partial^2 \psi}{\partial \eta^2} = n \frac{\partial p}{\partial \eta} = \frac{n}{(\partial \eta / \partial p)} = n\, p(1-p)$$

4. a) These are not complete since $E\, X_{(n)} - X_{(1)} = \frac{n-1}{n+1}$

b) $U = X_{(n)}/X_{(1)}$ is ancillary whereas $T = (X_{(1)}, X_{(n)})$ is sufficient.

Yet $EU = n\int_1^2 u(u-1)^{n-2} \underset{\text{parts}}{=} \frac{n-1}{n}$. Hence T is not complete

since $E\left[U - \frac{n-1}{n}\right] = 0$, yet $U \not\equiv \frac{n-1}{n}$